Sheet 1

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1. A pressure gauge with a range between 0 - 10 bar is found to have an error of ± 0.15 bar when calibrated by the manufacturer.

Calculate:

a. The error percentage of the gauge.

b. The error percentage when the reading obtained is 2.0 bar.

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2. Two pressure gauges (pressure gauge A and B) have a full scale accuracy of ± 5%. Sensor A has a range of 0-1 bar and Sensor B 0-10 bar. Which gauge is more suitable to be used if the reading is 0.9 bar?

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3. A pressure gauge of range 0-20 bar is said to have an error of ±0.25 bar when calibrated by the manufacturer. Calculate the percentage error on the basis of maximum scale value. What would be the possible error as a percentage of the indicated value when a reading of 5 bar is obtained in a test?

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4. A spring scale requires a change of $15kg_f$ in the applied weight to produce a 2 cm change in the deflection of the spring scale. Determine the static sensitivity.

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5. Explain the following statements:

(i) A galvanometer has a sensitivity specified of 15 mm/μA.

(ii) A Platinum Resistance Thermometer has a sensitivity specified of 0.5 $\Omega/^oC$

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6. The output of a platinum resistance thermometer (RTD) is as follows:

Input(°C)	Output(Ohm)
0	0
100	200
200	400
300	600
400	800

Calculate the sensitivity of the equipment.

7. A measuring system consists of a transducer, an amplifier and a recorder, and their individual sensitivities are stated as follows:

Transfer sensitivity K1 = 0.25 mV/ °C

Amplifier gain K2 = 2.5 V/mV

Recorder sensitivity K3 = 4 mm/V

What would be the overall sensitivity of the measuring system?

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8. A pressure measuring system consists of a piezoelectric transducer, a charge amplifier and a ultra violet charge recorder. The sensitivities of these elements are stated as follows :

Piezoelectric transducer, K1 = 8.5 PC/bar

Charge amplifier, K2 = 0.004 V/PC

Ultraviolet charge recorder, K3 = 20 mm/V

What would be the deflection on the chart due to a pressure change of 30 bar?

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9. **(a)** An instrument is calibrated in an environment at a temperature of 20°C and the following output readings y are obtained for various input values x:

y	13.1	26.2	39.3	52.4	65.5	78.6
x	5	10	15	20	25	30

Determine the measurement sensitivity, expressed as the ratio y/x.

(b) When the instrument is subsequently used in an environment at a temperature of 50°C, the input/output characteristic changes to the following:

y	14.7	29.4	44.1	58.8	73.5	88.2
x	5	10	15	20	25	30

Determine the new measurement sensitivity. Hence determine the sensitivity drift due to the change in ambient temperature of 30°C.

10. A load cell is calibrated in an environment at a temperature of 21°C and has the following deflection/load characteristic:

Load (kg)	0	50	100	150	200
Deflection (mm)	0	1	2	3	4

When used in an environment at 35°C, its characteristic changes to the following:

Load (kg)	0	50	100	150	200
Deflection (mm)	0.2	1.3	2.4	3.5	4.6

(a) Determine the sensitivity at 21°C and 35°C.

(b) Calculate the total zero drift and sensitivity drift at 35°C.

(c) Hence determine the zero drift and sensitivity drift coefficients (in units of μm/°C and (μm per kg)/(°C)).

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11. An unmanned submarine is equipped with temperature and depth measuring instruments and has radio equipment that can transmit the output readings of these instruments back to the surface. The submarine is initially floating on the surface of the sea with the instrument output readings in steady state. The depth measuring instrument is approximately zero order and the temperature transducer first order with a time constant of 50 seconds. The water temperature on the sea surface, T_o, is 20°C and the temperature T_x at a depth of x metres is given by the relation:

$$T_x = T_o - 0.01x$$

(a) If the submarine starts diving at time zero, and thereafter goes down at a velocity of 0.5 metres/second, draw a table showing the temperature and depth measurements reported at intervals of 100 seconds over the first 500 seconds of travel. Show also in the table the error in each temperature reading.

(b) What temperature does the submarine report at a depth of 1000 metres?

STATIC CHARACTERISTICS

The difference between the measured value (Vm) and the true value (Vt) of the quantity represents **static error** or **absolute error** of measurement (Es)

$$Es = Vm - Vt$$

The **static correction** is defined as *the difference between the true value and the measured value of a quantity.*

$$Cs = Vt - Vm$$

$$Cs = -Es$$

ACCURACY

The accuracy of an instrument is a measure of how close the output reading of the instrument is to the correct value.

- Accuracy is the ability of an instrument to show the exact reading.
- Always related to the extent of the wrong reading/non accuracy.
- Normally shown in percentage of error which of the full scale reading percentage.

Answer of P1:

a. $\text{Error Percentage} = \pm \dfrac{0.15\ bar}{10\ bar} \times 100 = \pm 1.5\ \%$

b. $\text{Error Percentage} = \pm \dfrac{0.15\ bar}{2\ bar} \times 100 = \pm 7.5\ \%$

The gauge is not suitable for use for low range reading.

Alternative : use gauge with a suitable range.

Answer of P2:

Sensor A :

$\text{Equipment max error} = \pm \dfrac{5}{100} \times 1\ bar = \pm 0.05\ bar$

Equipment accuracy

$0.9\text{ bar (in \%)} = \pm \dfrac{0.05\ bar}{0.9\ bar} \times 100 = \pm 5.6\ \%$

Sensor B :

$$\text{Equipment max error} = \pm\frac{5}{100}\times 10\ bar = \pm 0.5\ bar$$

Equipment accuracy

$$0.9 \text{ bar (in \%)} = \pm\frac{0.5\ bar}{0.9\ bar}\times 100 = \pm 55\ \%$$

Conclusion :

Sensor A is more suitable to use at a reading of 0.9 bar because the error percentage (± 5.6%) is smaller compared to the percentage error of Sensor B (± 55%).

Answer of P3:

$$\text{Pressure gauge max error} = \pm\frac{\text{Percentage error (accuracy)}}{100}\times 20\ bar = \pm 0.25\ bar$$

$$\text{Percentage error (accuracy)} = \pm\frac{0.25}{20}\times 100 = 1.25\ \%$$

Pressure gauge accuracy

$$5 \text{ bar (in \%)} = \pm\frac{0.25\ bar}{5\ bar}\times 100 = \pm 5\ \%$$

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SENSITIVITY

- Defined as the ratio of change in output towards the change in input at a steady state condition.

$$\text{Sensitivity (k)} = \frac{\Delta\theta_o}{\Delta\theta_i}$$

$\Delta\theta_o$: change in output; $\Delta\theta_i$: change in input

Example :

The resistance value of a Platinum Resistance Thermometer changes when the temperature increases. Therefore, the unit of sensitivity for this equipment is Ohm/°C.

Example :

Pressure sensor A with a value of 2 bar caused a deviation of 10 degrees. Therefore, the sensitivity of the equipment is 5 degrees/bar.

$$\text{Sensitivity (k)} = \frac{\textit{Scale deflection}}{\textit{Value of measurand producing deflection}}$$

Answer of P4:

$$\text{Sensitivity (k)} = \frac{\textit{Scale deflection}}{\textit{Value of measurand producing deflection}}$$

$$\text{Sensitivity (k)} = \frac{2\,cm}{15\,kg} = 1.333\ mm/kg$$

Answer of P6:

Draw an input versus output graph. From that graph, the sensitivity is the slope of the graph.

$$\text{Sensitivity (k)} = \frac{\Delta\theta_o}{\Delta\theta_i} = \frac{(400-200)\,ohm}{(200-100)^oC} = 2\ ohm/^oC$$

Sensitivity of the whole system is $k = k_1 \times k_2 \times \ldots \times k_n$

$$k_1 = \frac{\theta_1}{\theta_i} \qquad k_2 = \frac{\theta_2}{\theta_1} \qquad k_3 = \frac{\theta_o}{\theta_2}$$

$$\text{Overall Sensitivity (k)} = \frac{\theta_o}{\theta_i} = \frac{\theta_1}{\theta_i} \times \frac{\theta_2}{\theta_1} \times \frac{\theta_o}{\theta_2}$$

$$= k_1 \times k_2 \times k_3$$

Example:

Consider a measuring system consisting of a transducer, amplifier and a recorder, with sensitivity for each equipment given below:

Transducer sensitivity	0.2 mV/°C
Amplifier gain	2.0 V/mV
Recorder sensitivity	5.0 mV/V

***Therefore,* Sensitivity of the whole system:**

$$k = k_1 \times k_2 \times k_3 = \frac{0.2\,mV}{^oC} \times \frac{2V}{mV} \times \frac{5\,mV}{V} = 2\ mV/^oC$$

Answer of P7:

Sensitivity of the whole system:

$$k = k_1 \times k_2 \times k_3 = \frac{0.25\,mV}{^oC} \times \frac{2.5V}{mV} \times \frac{4\,mm}{V} = 2.5\ mm/^oC$$

Answer of P8:

Sensitivity of the whole system:

$$k = k_1 \times k_2 \times k_3 = \frac{8.5\,pC}{bar} \times \frac{0.004V}{pC} \times \frac{20\,mm}{V} = 0.68\ mm/bar$$

$$\because \ \text{Sensitivity (k)} = \frac{Scale\ deflection\ \ (mm)}{Value\ of\ measurand\ producing\ deflection\ (bar)}$$

$$\therefore \ \ deflection = 0.68 \times 30 = 20.4\ mm$$

Answer of P9:

At 20°C $\quad$ $\text{Sensitivity} = \dfrac{\Delta y}{\Delta x} = \dfrac{26.2 - 13.1}{10 - 5} = 2.62$

At 50°C $\quad$ $\text{Sensitivity} = \dfrac{\Delta y}{\Delta x} = \dfrac{29.4 - 14.7}{10 - 5} = 2.94$

$$\text{Sensitivity drift} = \text{Sensitivity}_{50} - \text{Sensitivity}_{20} = 2.94 - 2.62 = 0.32$$

Answer of P10:

a. At 21°C $\quad$ $\text{Sensitivity} = \dfrac{\Delta y}{\Delta x} = \dfrac{1 - 0}{50 - 0} = 20\ \mu m/kg$

At 35°C $\quad$ $\text{Sensitivity} = \dfrac{\Delta y}{\Delta x} = \dfrac{1.3 - 0.2}{50 - 0} = 22\ \mu m/kg$

b. $\text{Zero drift} = 0.2\ mm = 200\ \mu m$

$$\text{Sensitivity drift} = \text{Sensitivity}_{35} - \text{Sensitivity}_{21} = 22 - 20 = 2\ \mu m/kg$$

c. $\text{Zero drift}/^oC = \dfrac{200}{14} = 14.2857\ \mu m/^oC$

$$\text{Sensitivity drift}/^oC = \frac{2}{14} = 0.1429\ (\mu m\ per\ kg)/^oC$$

DYNAMIC CHARACTERISTICS

A general measurement system can be mathematically described by the .following differential equation :

$$a_n \frac{d^n q_0}{dt^n} + a_{n-1}\frac{d^{n-1} q_0}{dt^{n-1}} + \cdots + a_1 \frac{dq_0}{dt} + a_0 q_0$$

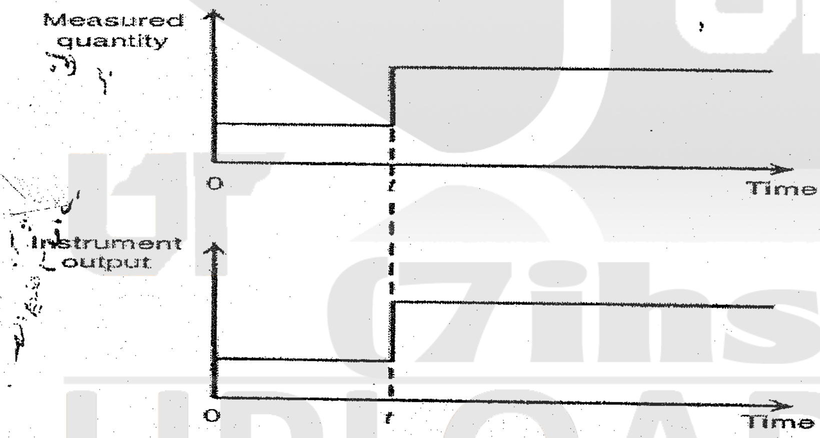
$$= b_m \frac{d^m q_i}{dt^m} + b_{m-1}\frac{d^{m-1} q_i}{dt^{m-1}} + \cdots + b_1 \frac{dq_i}{dt} + b_0 q_i$$

Zero order instrument:

If all the coefficients $a_1 \ldots a_n$ other than a_o are assumed zero, then

$$a_o q_o = b_o q_i \longrightarrow q_o = b_o q_i / a_o = kq_i$$

where K is the sensitivity of the system.

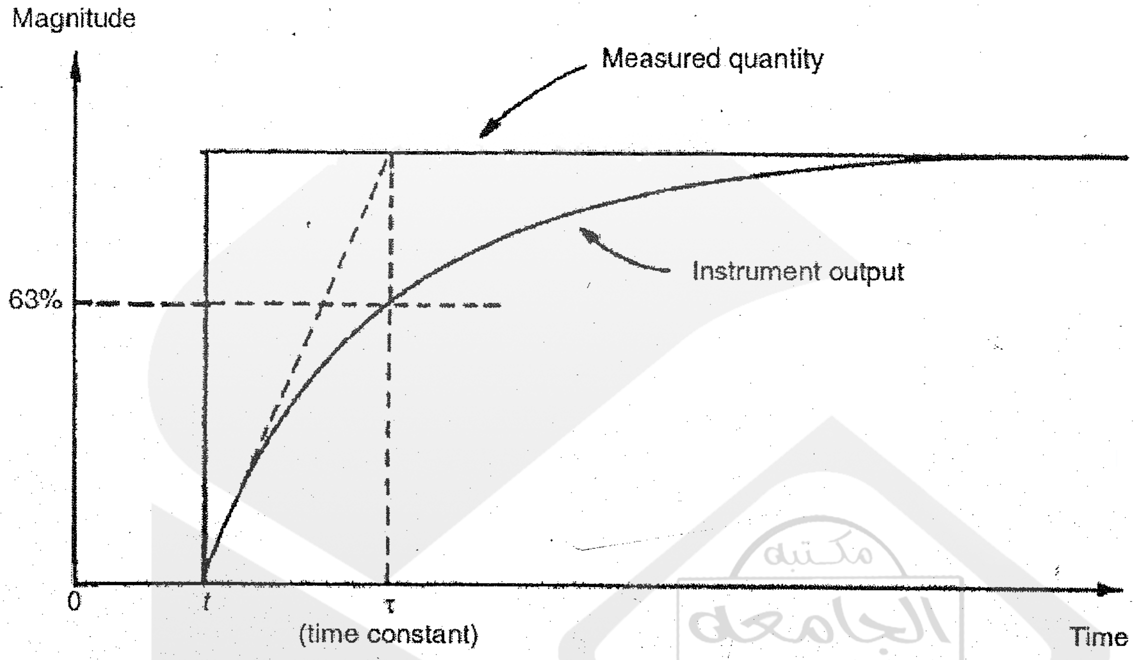


First order instrument: The behavior of a first-order system is represented by a first-order differential equation of the form

$$a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i$$

$$q_o = \frac{(b_o / a_o) q_i}{[1 + (a_1 / a_o) D]} \qquad\qquad q_o = \frac{k\, q_i}{1 + \tau D}$$

where 'τ' is the time constant ($\tau = a_1 / a_0$) and 'K' is the static sensitivity ($K = b_0 / a_0$)



Second-order instrument:

$$a_2 \frac{d^2 q_0}{dt^2} + a_1 \frac{dq_0}{dt} + a_0 q_0 = b_0 q_i$$

$$q_o = \frac{(b_o / a_o) q_i}{[1 + (a_1 / a_o) D + (a_2 / a_o) D^2]}$$

$$\frac{q_o}{q_i} = \frac{k}{[1 + 2\zeta D / \omega + D^2 / \omega^2]}$$