

# Kinematics of A particle

## Position and Displacement:

$$\vec{r} = \vec{s} = f(t) \quad m$$

## Velocity:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

m/sec

## Acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

m/sec<sup>2</sup>

$$\vec{a} = \frac{d\vec{v}}{d\vec{r}} \left( \frac{d\vec{r}}{dt} \right)$$

$$\vec{a} = \vec{v} \frac{d\vec{v}}{d\vec{r}}$$

derivative

integration



$$\vec{V} = \vec{V}_0 + \int_0^t \vec{a} dt$$

$$\vec{a} = \frac{d\vec{V}}{dt}$$

$$d\vec{V} = \vec{a} dt$$

$$\int_{V_0}^V d\vec{V} = \int_0^t \vec{a} dt$$

$$\vec{V} \Big|_{V_0}^V = \int_0^t \vec{a} dt$$

$$\vec{V} - \vec{V}_0 = \int_0^t \vec{a} dt$$

$$\vec{r} = \vec{r}_0 + \int_0^t \vec{V} dt$$

$$\vec{a} = \vec{V} \frac{d\vec{V}}{d\vec{r}}$$

$$\int_{\vec{r}_0}^{\vec{r}} \vec{a} d\vec{r} = \int_{V_0}^V \vec{V} dV$$

$$\vec{V} = V_0 + \int_0^t \vec{a} dt$$

Newton's law  $\Rightarrow$   $a$  is constant  $\vec{V} = \frac{d\vec{r}}{dt}$

$$V = V_0 + at \rightarrow (1)$$

$$d\vec{r} = \vec{V} dt$$

$$\int_{\vec{r}_0}^{\vec{r}} d\vec{r} = \int_0^t \vec{V} dt$$

$$r = r_0 + V_0 t + \frac{1}{2} at^2 \rightarrow (2)$$

$$V^2 = V_0^2 + 2a(r - r_0) \rightarrow (3)$$

$$\vec{r} \Big|_{r_0}^{\vec{r}} = \int_0^t \vec{V} dt$$

$$\vec{r} - \vec{r}_0 = \int_0^t \vec{V} dt$$

$$\vec{r} = \vec{r}_0 + \int_0^t \vec{V} dt$$

# ① Rectangular Coordinates (Cartesian Coordinates): $x, y, z$

## ⊗ Position vector

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

displacement (distance)

$$r = \sqrt{x^2 + y^2 + z^2}$$

## ⊗ Velocity vector

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k}$$

magnitude of velocity  $\|\vec{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

## ⊗ acceleration vector

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x(t)\hat{i} + a_y(t)\hat{j} + a_z(t)\hat{k}$$

$$\|\vec{a}\| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

↳ magnitude of acceleration.

$$\vec{a} = \vec{v} \frac{d\vec{v}}{d\vec{r}}$$

(4)

$$\vec{V} = \vec{V}_0 + \int_0^t \vec{a} dt \quad \#$$

$$\vec{r} = \vec{r}_0 + \int_0^t \vec{V} dt \quad \#$$

$$\int_{V_0}^V \vec{V} dV = \int_{r_0}^r \vec{a} dr \quad \#$$

Determine the Cartesian eq<sup>n</sup> of the path

↓

$$y = P(x)$$

↓

$$\text{Trajectory eq<sup>n</sup>}$$

(5)

EX 1.48.

$$\vec{r} = (5 + 3\cos t)\hat{i} + (4\sin t)\hat{j}$$

① Trajectory eq = ②  $v = ?$  &  $a = ?$  @  $t = 0$ 

Sol

$$① \quad x = 5 + 3\cos t$$

$$y = 4\sin t$$

$$\cos t = \frac{x-5}{3}$$

$$\sin t = \frac{y}{4}$$

$$\cos^2 t = \frac{(x-5)^2}{9}$$

$$\sin^2 t = \frac{y^2}{16}$$

$$\frac{(x-5)^2}{9} + \frac{y^2}{16} = \cos^2 t + \sin^2 t$$

$$\left\{ \frac{(x-5)^2}{9} + \frac{y^2}{16} = 1 \right\}$$

$$② \quad \vec{v} = \frac{d\vec{r}}{dt} = (-3\sin t)\hat{i} + (4\cos t)\hat{j} \quad \text{m/sec}$$

$$③ \quad \vec{a} = \frac{d\vec{v}}{dt} = (-3\cos t)\hat{i} + (-4\sin t)\hat{j} \quad \text{m/sec}^2$$



(6)

@t=0

$$\vec{v} = 0\hat{i} + 4\hat{j} \Rightarrow \|\vec{v}\| = \sqrt{0^2 + 4^2} = 4 \text{ m/sec}$$

$$\vec{a} = -3\hat{i} + 0\hat{j} \Rightarrow \|\vec{a}\| = \sqrt{(-3)^2 + (0)^2} = 3 \text{ m/sec}^2$$

Ex 1.5: Prob (23)

$$\vec{v} = 12t^2\hat{i} + 16t^3\hat{j} + \sin \pi t \hat{k}$$

$$\vec{r}_0 = 4\hat{j} + 3\hat{k} \text{ m} \quad r_0(0, 4, 3) \text{ m}$$

$$\vec{a} = ? \quad \vec{r} = ?$$

Sol

$$\vec{a} = \frac{d\vec{v}}{dt} = 24t\hat{i} + 48t^2\hat{j} + \pi \cos \pi t \hat{k}$$

$$\vec{r} = \vec{r}_0 + \int_0^t \vec{v} dt$$

$$\vec{r} = 4\hat{j} + 3\hat{k} + \int_0^t [(12t^2)\hat{i} + (16t^3)\hat{j} + (\sin \pi t)\hat{k}] dt$$

$$\vec{r} = 4\hat{j} + 3\hat{k} + \left[ \frac{12t^3}{3}\hat{i} + \frac{16t^4}{4}\hat{j} - \frac{1}{\pi} \cos \pi t \hat{k} \right]_0^t \quad (7)$$

$$\vec{r} = 4\hat{j} + 3\hat{k} + 4t^3\hat{i} + 4t^4\hat{j} + \left( -\frac{1}{\pi} \cos \pi t + \frac{1}{\pi} \right) \hat{k}$$

$$\vec{r} = 4t^3\hat{i} + (4t^4 + 4)\hat{j} + \left( -\frac{1}{\pi} \cos \pi t + \frac{1}{\pi} + 3 \right) \hat{k}$$

another sol<sup>n</sup>

$$V_x = 12t^2$$

$$V_y = 16t^3$$

$$V_z = \sin \pi t$$

$$x = x_0 + \int_0^t V_x dt$$

$$y = y_0 + \int_0^t V_y dt$$

$$z = z_0 + \int_0^t V_z dt$$

$$x = 0 + \int_0^t 12t^2 dt$$

$$y = 4 + \int_0^t 16t^3 dt$$

$$z = 3 + \int_0^t \sin \pi t dt$$

$$x = 4t^3$$

$$y = 4 + 4t^4$$

$$z = 3 - \left( \frac{1}{\pi} \cos \pi t \right) \Big|_0^t$$

$$z = 3 - \frac{1}{\pi} \cos \pi t + \frac{1}{\pi}$$

$$\vec{r} = 4t^3\hat{i} + (4t^4 + 4)\hat{j} + \left( -\frac{1}{\pi} \cos \pi t + \frac{1}{\pi} + 3 \right) \hat{k}$$

Prob (14) :

$$\vec{r} = (5 \sin t + 3) \hat{i} + 5 \cos t \hat{j}$$

Soln

$$x = 5 \sin t + 3$$

$$y = 5 \cos t$$

$$\sin t = \frac{x-3}{5}$$

$$\cos t = \frac{y}{5}$$

$$\sin^2 t = \frac{(x-3)^2}{25}$$

$$\cos^2 t = \frac{y^2}{25}$$

$$\frac{(x-3)^2}{25} + \frac{y^2}{25} = 1$$

$$\vec{v} = \frac{d\vec{r}}{dt} = (5 \cos t) \hat{i} + (-5 \sin t) \hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -5 \sin t \hat{i} - 5 \cos t \hat{j}$$

$$@ t=1 \quad \vec{v} =$$

$$\vec{a} =$$

$$@ t=3 \quad \vec{v} =$$

$$\vec{a} =$$



(9)

Prob (19):

$$\vec{a} = 9t^2 \hat{i} + 12t^3 \hat{j} - 6t \hat{k}$$

$$\vec{r} = ? \text{ @ } t = 2 \text{ sec}$$

$$\text{@ } t=0 \Rightarrow \vec{r}_0 = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{v}_0 = 2\hat{i} - 6\hat{j} + 5\hat{k}$$

Sol

$$\vec{v} = \vec{v}_0 + \int_0^t \vec{a} \, dt$$

$$\vec{v} = 2\hat{i} - 6\hat{j} + 5\hat{k} + \int_0^t (9t^2 \hat{i} + 12t^3 \hat{j} - 6t \hat{k}) \, dt$$

$$\vec{v} = 2\hat{i} - 6\hat{j} + 5\hat{k} + (3t^3 \hat{i} + 3t^4 \hat{j} - 3t^2 \hat{k}) \Big|_0^t$$

$$\boxed{\vec{v} = (3t^3 + 2)\hat{i} + (3t^4 - 6)\hat{j} + (-3t^2 + 5)\hat{k}}$$

$$\vec{r} = \vec{r}_0 + \int_0^t \vec{v} \, dt$$

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \int_0^t ((3t^3 + 2)\hat{i} + (3t^4 - 6)\hat{j} + (-3t^2 + 5)\hat{k}) \, dt$$

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \left( \frac{3}{4}t^4 + 2t \right) \hat{i} + \left( \frac{3}{5}t^5 - 6t \right) \hat{j} + \left( -t^3 + 5t \right) \hat{k} \Big|_0^t$$

$$\vec{r} = \left( \frac{3}{4}t^4 + 2t + 2 \right) \hat{i} + \left( \frac{3}{5}t^5 - 6t - 1 \right) \hat{j} + \left( -t^3 + 5t + 2 \right) \hat{k}$$

(10)

① t = 2

$$\vec{r} = 18\hat{i} + \left(\frac{96}{5} - 12 - 1\right)\hat{j} + (-8 + 10 + 2)\hat{k}$$

$$r = \sqrt{(\hat{i})^2 + (\hat{j})^2 + (\hat{k})^2}$$

Prob (20) :-

$$y = x^2$$

$$V_y = 3 \text{ m/sec}$$

$$\|V\| = ? \text{ @ } t = 2$$

$$\text{@ } t = 0 \Rightarrow r_0 = (1, 1)$$

$$\|r\| = ? \text{ @ } t = 2$$

$$y = y_0 + \int_0^t V_y dt$$

$$y = 1 + \int_0^t 3 dt = (1 + 3t)$$

$$y = x^2 \Rightarrow x = y^{1/2}$$

$$x = (1 + 3t)^{1/2}$$

$$\vec{r} = (1 + 3t)^{1/2} \hat{i} + (1 + 3t) \hat{j}$$

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{1}{2}(1 + 3t)^{-1/2} (3) \hat{i} + 3 \hat{j}$$

(11)

$$@ t=2 \quad \vec{r} = \sqrt{7} \hat{i} + 7 \hat{j}$$

$$\vec{v} = \frac{3}{2\sqrt{7}} \hat{i} + 3 \hat{j}$$

$$\|\vec{r}\| = \sqrt{(\sqrt{7})^2 + (7)^2} = \quad \text{m}$$

$$\|\vec{v}\| = \sqrt{\left(\frac{3}{2\sqrt{7}}\right)^2 + (3)^2} = \quad \text{m/sec}$$

prob (25)

↑ ↑ ↑ ↑

$$v_x = 5t$$

$$y = 0.5x^2$$

||a||

$$\|\vec{r}\| = ?$$

$$@ t=0 \Rightarrow \begin{matrix} x=0 \\ y=0 \end{matrix}$$

$$@ t=1 \text{ sec}$$

$$x = x_0 + \int_0^t v_x dt$$

Soln

$$x = 0 + \int_0^t 5t dt$$

$$x = \frac{5t^2}{2}$$

$$\Rightarrow y = 0.5(2.5t^2)^2$$

$$y = 3.125t^4$$

$$\vec{r} = 2.5t^2 \hat{i} + 3.125t^4 \hat{j}$$

(12)

$$\vec{v} = \frac{d\vec{r}}{dt} = 5t \hat{i} + 12.5t^3 \hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 5 \hat{i} + 37.5t^2 \hat{j}$$

$$\textcircled{a} \text{ } t=1 \quad \vec{r} = 2.5 \hat{i} + 3.125 \hat{j}$$

$$\vec{v} = 5 \hat{i} + 12.5 \hat{j}$$

$$\vec{a} = 5 \hat{i} + 37.5 \hat{j}$$

$$\|\vec{r}\| = \sqrt{(2.5)^2 + (3.125)^2} = \quad \text{m}$$

$$\|\vec{v}\| = \sqrt{(5)^2 + (12.5)^2} = \quad \text{m/sec}$$

$$\|\vec{a}\| = \sqrt{(5)^2 + (37.5)^2} = \quad \text{m/sec}^2$$

Prob 1.21

$$x = 5 \cos 2t$$

$$y = 5 \sin 2t$$

$$z = 3t$$

$$\vec{a} = ?$$

Sol

$$\vec{r} = 5 \cos 2t \hat{i} + 5 \sin 2t \hat{j} + 3t \hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -10 \sin 2t \hat{i} + 10 \cos 2t \hat{j} + 3 \hat{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -20 \cos 2t \hat{i} - 20 \sin 2t \hat{j}$$

H.W Prob 16

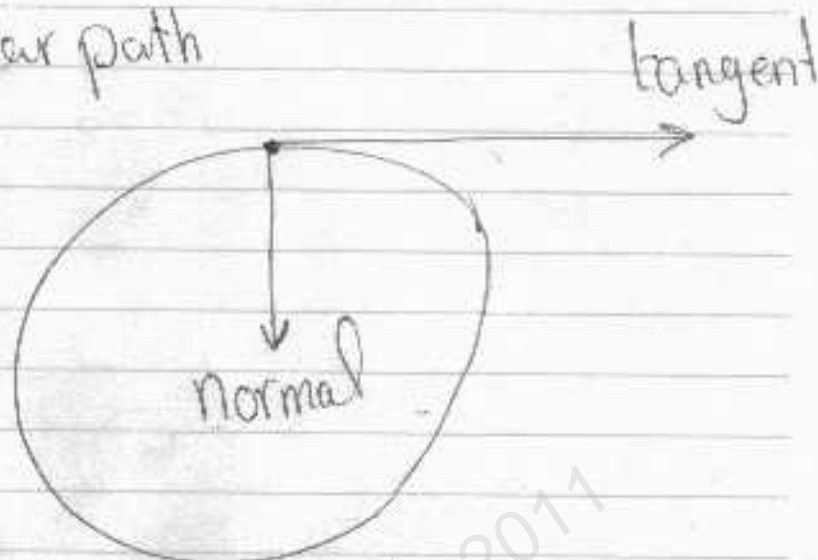
18

12

# ⑧ Natural Coordinates

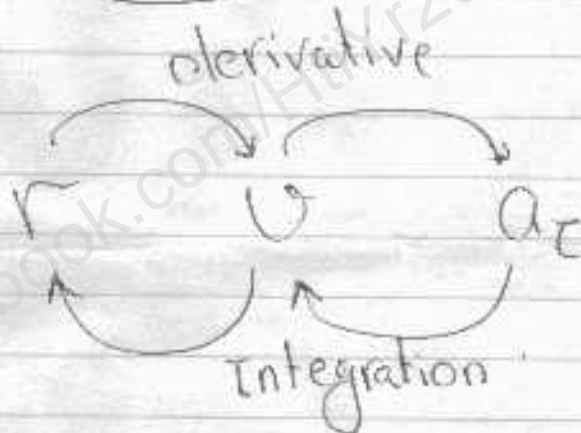
⑭

Curve, Circular path



$$S' = r = P(t)$$

$$v = \frac{dr}{dt}$$



$$a_t = \frac{dv}{dt}$$

↳ tangent acceleration

$$a_n = \frac{v^2}{\rho}$$

↳ normal acceleration

$$a_t = \sqrt{a_t^2 + a_n^2}$$

$$a_t = \frac{dv}{dr} \left( \frac{dr}{dt} \right) \Rightarrow a_t = v \frac{dv}{dr}$$



prob 1.47:

$$R = 300 \text{ m}$$

$$15 \text{ m/sec} \xrightarrow{\text{in 3 sec}} 27 \text{ m/sec}$$

$$a_t = ?$$

$$v = 20 \text{ m/sec}$$

Soln

$$a_t = \frac{dv}{dt}$$

$$a_t = \frac{27 - 15}{3}$$

$$a_n = \frac{v^2}{R}$$

$$a_n = \frac{(20)^2}{300} = \frac{4}{3} \text{ m/sec}^2$$

$$a_t = 4 \text{ m/sec}^2$$

$$a_t = \sqrt{a_t^2 + a_n^2}$$

$$a_t = \sqrt{(4)^2 + \left(\frac{4}{3}\right)^2} = \quad \text{m/sec}^2$$

prob 1.49:

$$R = 100 \text{ m}$$

$$v = 3(t + t^2) = 3t + 3t^2$$

$$a = ? \text{ @ } t = 3 \text{ sec}$$

$$r = ? \text{ @ } t = 3 \text{ sec}$$

Soln

$$a_t = \frac{dv}{dt}$$

$$a_n = \frac{v^2}{R}$$

$$a_t = 3 + 6t$$

$$a_n = \frac{(3t + 3t^2)^2}{100}$$

$$\text{@ } t = 3 \text{ sec}$$

$$a_t = 3 + 6 \times 3 = 21 \text{ m/sec}^2$$

$$a_n = \frac{(3(3) + 3(3)^2)^2}{100} = 12.96 \text{ m/sec}^2$$

$$a_t = \sqrt{a_t^2 + a_n^2}$$

$$= \sqrt{(21)^2 + (12.96)^2} = \text{m/sec}^2$$

(17)

$$r = r_0 + \int_0^t v \, dt$$

$$r = \int_0^t (3t + 3t^2) \, dt$$

$$r = \frac{3t^2}{2} + \frac{3t^3}{3}$$

$$@ t = 3 \text{ sec}$$

$$r = \frac{3(3)^2}{2} + \frac{3(3)^3}{3} = 40.5 \text{ m}$$

Prob 1.54:-

$$R = 300 \text{ m}$$

$$a_r = 0.75 \text{ m/sec}^2$$

$$a_t = 0.9 \text{ m/sec}^2$$

$$r = ?$$

$$t = ?$$

Soln

$$v = v_0 + \int_0^t a_r \, dt$$

$$v = \int_0^t 0.75 \, dt$$

$$a_t = \sqrt{a_r^2 + a_n^2}$$

$$(0.9)^2 = (0.75)^2 + a_n^2$$

$$a_n = 0.497 \text{ m/sec}^2$$

$$V = 0.75t \quad \text{--- (1)}$$

$$r = \int_0^t V dt$$

$$r = \int_0^t 0.75t dt$$

$$r = \frac{0.75t^2}{2} = 0.375t^2 \quad \text{--- (2)}$$

From (1), (3) we get

$$0.75t = 12.2$$

$$t = 16.28 \text{ sec}$$

Sub in eq. (2)

$$r = 0.375(16.28)^2$$

$$r = 99.5 \text{ m}$$

$$a_n = \frac{V^2}{\rho}$$

$$0.497 = \frac{V^2}{300}$$

$$V = 12.2 \text{ m/sec} \quad \text{--- (3)}$$

Prob 1.52 :

$$R = 350 \text{ m}$$

$$V = 72 \text{ km/h}$$

$$V = 20 \text{ m/sec}$$

$$a_r = -1.25 \text{ m/sec}^2$$

$$a_t = ? \quad \begin{array}{l} \nearrow \text{a) immediately} \\ \searrow \text{b) 45' later} \end{array}$$

Sol<sup>n</sup>

a)  $a_r = -1.25 \text{ m/sec}^2$

$$a_n = \frac{V^2}{R} = \frac{(20)^2}{350} = 1.143 \text{ m/sec}^2$$

$$a_t = \sqrt{a_r^2 + a_n^2} = 1.694 \text{ m/sec}^2$$

b)  $V = V_0 + \int_0^t a_t dt$

$$V = \underline{20} + \int_0^t \dots dt$$

$$V = 20 - 1.25t$$

@  $t = 4 \text{ sec}$

$$V = 20 - 1.25 \times 4 = 15 \text{ m/sec}$$

$$C_n = \frac{V^2}{r} = \frac{(15)^2}{350} = 0.643 \text{ m/sec}^2$$

$$a_t = \sqrt{a_r^2 + a_n^2}$$

$$a_t = \sqrt{(-1.25)^2 + (0.643)^2} = \underline{\underline{1.41 \text{ m/sec}^2}}$$

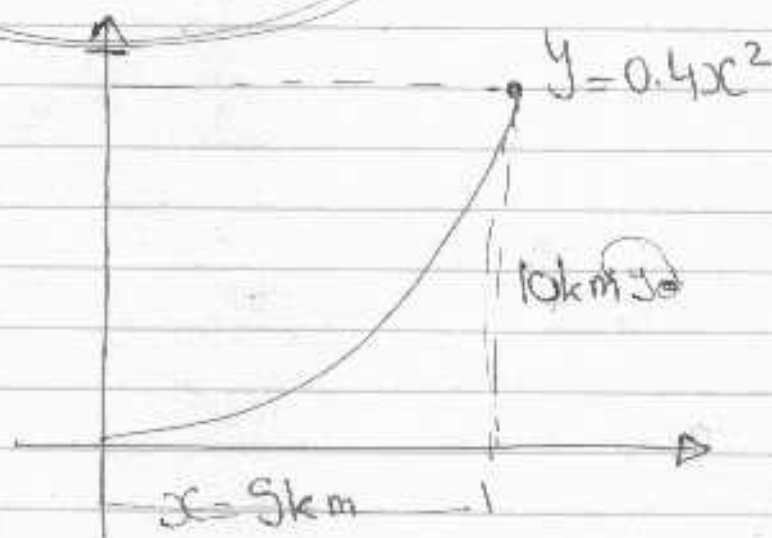
Prob 1.50

$$V = 200 \text{ m/sec}$$

Constant speed

$$a = ?$$

$$\frac{v(1+y'^2)^{3/2}}{y''}$$





$$y = 0.4x^2$$

$$y' = 0.8x$$

$$y'' = 0.8$$

$$\Rightarrow \rho = \frac{(1 + (0.8x)^2)^{3/2}}{0.8}$$

$$\rho = \frac{(1 + (0.8(5))^2)^{3/2}}{0.8}$$

$$\boxed{\rho = 87.6 \text{ km}}$$

$$a_t = 0$$

$$a_n = \frac{v^2}{\rho}$$

$$= \frac{(200)^2}{(87.6 \times 10^3)}$$

$$= 0.46 \text{ m/sec}^2$$

$$a_t = \sqrt{a_{t0}^2 + a_n^2}$$

$$= 0.46 \text{ m/sec}^2$$

Prob 1.48

22

$$R = 50 \text{ m}$$

$$V_0 = 4 \text{ m/sec} \Rightarrow S' = 0$$

$$V' = 0.05 S'$$

$$V = ?$$

$$a = ? \Rightarrow S' = 10 \text{ m}$$

Soln

$$V' = 0.05 S'$$

$$\frac{dV}{dt} = 0.05 S'$$

$$\frac{dV}{dS} \left( \frac{dS}{dt} \right) = 0.05 S'$$

$$V \frac{dV}{dS} = 0.05 S'$$

$$V dV = 0.05 S' dS$$

$$\int_{V_0}^V V dV = \int_{S_0}^{S'} 0.05 S' dS$$

$$V_0 = 4$$

$$S_0 = 0$$

$$\frac{V^2}{2} \Big|_4^V = \frac{0.05 S'^2}{2} \Big|_0^S \quad (23)$$

$$\frac{V^2}{2} - 8 = \frac{0.05 S'^2}{2} - 0$$

$$V^2 - 16 = 0.05 S'^2$$

$$V^2 = 0.05 S'^2 + 16$$

$$V = \sqrt{0.05 S'^2 + 16}$$

$$\text{@ } S' = 10 \quad V = \sqrt{0.05(10)^2 + 16} = \sqrt{21} \text{ m/sec}$$

$$a_n = \frac{V^2}{\rho} = \frac{(\sqrt{21})^2}{50} = 0.42 \text{ m/sec}^2$$

$$a_t = \frac{dV}{dt} = 0.05 S'$$

$$a_t = 0.05(10) = 0.5 \text{ m/sec}^2$$

$$a_t = \sqrt{a_t^2 + a_n^2} = \boxed{0.653} \text{ m/sec}^2$$

prob(7)

$$v = \frac{5}{4+s'}$$

$$a = ? \quad @ s' = 2$$

Soln

$$a = v \frac{dv}{ds'}$$

$$a = \frac{5}{4+s'} \left( \frac{(4+s')(0) - 5(1)}{(4+s')^2} \right)$$

$$a = \frac{-25}{(4+s')^3}$$

$$@ s' = 2$$

$$a = \frac{-25}{(4+2)^3} = \frac{-25}{216} = \underline{\underline{-0.116 \text{ m/sec}^2}}$$

prob(5)

$$a = 2t - 9$$

$$@ t = 0 \Rightarrow s_0 = 1 \text{ m}$$

$$v_0 = 10 \text{ m/sec}$$

@  $t=10 \Rightarrow$  (a)  $s'$

(b)  $U$

(c)  $a$

$$U = U_0 + \int_0^t a \, dt$$

$$U = 10 + \int_0^t (2t - g) \, dt$$

$$\boxed{U = 10 + t^2 - gt}$$

$$S' = S_0 + \int_0^t U \, dt$$

$$S' = 1 + \int_0^t (t^2 - gt + 10) \, dt$$

$$\boxed{S' = 1 + \frac{t^3}{3} - \frac{gt^2}{2} + 10t}$$

@  $t = 10 \text{ sec}$        $S' =$        $\text{m}$

$U =$        $\text{m/sec}$

$a =$        $\text{m/sec}^2$

Prob 1.68.

26

$$a = 60 v^{-4}$$

$$v_0 = 2 \text{ m/sec}$$

$$v = ? \text{ @ } t = 3 \text{ sec}$$

$$s = ?$$

Soln

$$a = 60 v^{-4}$$

$$\frac{dv}{dt} = 60 v^{-4}$$

$$\frac{1}{v^4} dv = 60 dt$$

$$\int_{v_0=2}^v v^4 dv = \int_0^t 60 dt$$

$$\frac{v^5}{5} \Big|_2^v = 60t \Big|_0^t$$

$$\frac{v^5}{5} - \frac{32}{5} = 60t$$



$$\frac{v^5}{5} = 60t + 6.4$$

$$v^5 = 300t + 32$$

$$v = (300t + 32)^{1/5}$$

$$s = \cancel{s_0} + \int_0^t v \, dt$$

$$s = \frac{1}{300} \int_0^t (300t + 32)^{1/5} \, dt$$

$$= \frac{1}{300} \left[ \frac{(300t + 32)^{6/5}}{6/5} \right] \Big|_0^t$$

$$s = \frac{1}{360} \left[ (300t + 32)^{6/5} - 64 \right]$$

@  $t = 3 \text{ sec}$  ,  $s =$  m

$v =$  m/sec

Prob 1.48

$$v = \frac{4}{a}$$

$$v = 4 \text{ m/sec} \rightarrow @ t = 2$$

$$a = ? \rightarrow @ t = 3 \text{ sec}$$

Soln

$$a = \frac{4}{v}$$

$$\frac{dv}{dt} = \frac{4}{v}$$

$$v dv = 4 dt$$

$$\int_4^v v dv = \int_2^t 4 dt$$

$$\frac{v^2}{2} \Big|_4^v = 4t \Big|_2^t$$

$$\frac{v^2}{2} - 8 = 4t - 8$$

$$\frac{v^2}{2} = 4t$$

$$v^2 = 8t$$

$$\boxed{v = \sqrt{8t}} \quad \boxed{v = \sqrt{8} t^{1/2}}$$

$$a = \frac{dv}{dt}$$

$$a = \frac{\sqrt{8}}{2} t^{-1/2} \quad |$$

$$@ t = 3 \text{ sec}$$

$$a = \frac{\sqrt{8}}{2} (3)^{-1/2} = \underline{\underline{\text{m/Sec}^2}}$$

prob 1.18:

$$V = 24 \text{ m/Sec}$$

$$S = 150 \text{ m}$$

$$a = ?$$

$$t = ?$$

Soln

$$V = V_0 + at$$

$$V^2 = V_0^2 + 2a(S - S_0)$$

$$(24)^2 = 2a(150) \Rightarrow \left( a = \text{m/Sec}^2 \right) |$$

$$24 = 0 + 1.92t \Rightarrow \left( t = 12.5 \text{ Sec} \right) |$$