UNIT - I

INTRODUCTION

Classification of systems :

1. Continuous - time system $y(t) = T[x(t)]$
2. Discrete - time system $y(n) = T[x(n)]$

Classification of Discrete - Time systems

(i) Static $y(n) = a x(n)$ (memory less)

(ii) Dynamic $y(n) = x(n+1) + x(n)$ (have memory)

(iii) Causal (o/p depends only present & past i/p not future inputs)

eg: $y(n) = x(n) + x(n-1)$

→ Non causal (s/y depends on future i/p s)

eg: $y(n) = x(2n)$

(iv) Linear

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

→ Non Linear

$$T[a_1 x_1(n) + a_2 x_2(n)] \neq a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

(v) Time invariant (or) shift invariant

↳ If the characteristics of the system do not change with time.

eg: $y(n,k) = y(n-k)$

→ Time variant

$$y(n,k) \neq y(n-k)$$

vi) stable :

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Classification of signals :

→ Based on the condition of independent variable :

(i) continuous Time signal (The i/p sgnl is continuously varied depending upon the time)

(ii) Discrete Time signal

↳ The sgnl is varied at any particular time interval .

→ Based on characteristics sgnl categorized by

(i) Analog signal

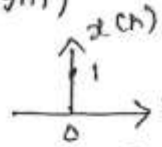
↳ Both dependent and independent variables are continuous

(ii) Digital signal

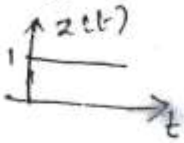
↳ Both dependent and independent variables are Discrete .

→ Basic signals : (Elementary continuous time sgnl)

(i) unit impulse signal ⇒ $\delta(n) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$



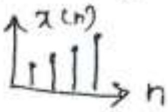
For continuous $\delta(t) = \begin{cases} 1 & t=0 \\ 0 & t \neq 0 \end{cases}$



(ii) Ramp signal

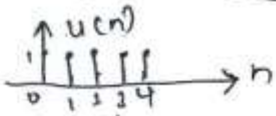
For DT $r(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$

For CT $r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$

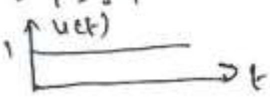


(iii) Unit step signal

$u(n) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$



$u(t) = \begin{cases} 1 & t=0 \\ 0 & t \neq 0 \end{cases}$



(iv) Exponential signal

$x(t) = e^{\alpha t}$

Rising Exponential

Classification of Discrete Time Signal:

(i) Energy and Power signals:

⇒ For a discrete-time signal $x(n)$ the energy E is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

→ Power is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

(ii) Periodic and aperiodic signal:

Periodic ⇒ $x(n+N) = x(n)$

→ For $x(n)$ the above eqn satisfies if and only if $\omega_0 N$ is an integer multiple of 2π

$$\omega_0 = 2\pi\left(\frac{m}{N}\right), \quad N = 2\pi\left(\frac{m}{\omega_0}\right)$$

Aperiodic ⇒ $x(n+N) \neq x(n)$

(iii) Symmetric (even) and Antisymmetric (odd) signal:

Symmetric (even) ⇒ $x(-n) = x(n)$ for all n

Antisymmetric (odd) ⇒ $x(-n) = -x(n)$ for all n.

(iv) Causal and Noncausal signal:

Causal → $x(n)$ is said to be causal if its value is zero for $n<0$.

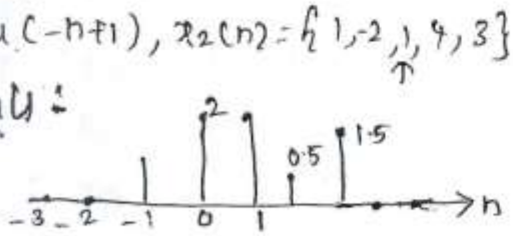
eg: $x_1(n) = a^n u(n)$, $x_2(n) = \{\underset{\uparrow}{1}, 2, -3, -1, 2\}$

Non causal → A sgnl that is zero for all $n \geq 0$

eg: $x_1(n) = a^n u(-n+1)$, $x_2(n) = \{1, -2, \underset{\uparrow}{1}, 4, 3\}$

Mathematical Representation of signals:

1. Graphical Representation



2. Functional Representation

$$x(n) = \begin{cases} 1 & \text{for } n=-1 \\ 2 & \text{for } n=0,1 \\ 0.5 & \text{for } n=2 \\ 1.5 & \text{for } n=3 \\ 0 & \text{otherwise} \end{cases}$$

3. Tabular Representation

n	-1	0	1	2	3
x(n)	1	2	2	0.5	1.5

4. Sequence Representation

$x(n) = \{1, \underset{\uparrow}{2}, 2, 0.5, 1.5\}$

Spectral Density :

Operation on signals :

The mathematical Transformation from one sgnl to another represented as $y(n) = T[x(n)]$

(i) shifting :
$y(n) = x(n-k)$ If k is +ive, the shifting delays the seq
If k is −ive, the shifting advance the seq

(ii) Time Reversal :
→ The time reversal of seq $x(n)$ can be obtained by folding the seq about $n=0$ i.e. $x(-n)$
eg: $x(-n+2)$ ⇒ is $x(-n)$ delayed by 2 units of time
$x(-n-2)$ ⇒ is $x(-n)$ advanced by 2 units of time

(iii) Time scaling :
↳ This is accomplished by replacing n by λn in the sequence $x(n)$ ∴ $x(n) = x(\lambda n)$, λ → scaled fact

(iv) scalar multiplier :
↳ If $x(n)$ is multiplied by scalar 'a' then the output of $y(n) = a \cdot x(n)$.

(v) signal multiplier : $x_1(n) \to \otimes \to y(n) = x_1(n)\, x_2(n)$, with $x_2(n)$ into $\otimes$

(vi) Addition operation : $x_1(n) \to \oplus \to y(n) = x_1(n) + x_2(n)$, with $x_2(n)$ into $\oplus$

sampling techniques :

→ sampling is Performed by applying a continuous time signal to an ADC whose o/p is a series of digital values.

$$x(nT) = x(n) \qquad -\infty < n < \infty$$

sample rate : The no. of samples per second

sample Period : The successive samples per second

sample Freq : Reciprocal of sampling period $F = 1/T$.

Quantization :

The Process of converting a discrete time continuous amp signal $x(n)$ into a discrete time amp sgnl $x_q(n)$.

→ This is done by rounding off each sample in $x(n)$ to the nearest quantization Level $q = \frac{\text{range of sgnl}}{\text{No. of quantization level}} = \frac{R}{2^b+1}$

Quantization Error : A quantization error arises when a continuous signal is converted into digital value.

$e(n) = x_q(n) - x(n)$

Nyquist rate :

A Continuous time sgnl can be completely represented in its samples and recovered back if and only if the sampling freq $f_s \geq 2f_{max}$

Aliasing Effect : $f_s \leq 2f_{max}$.

Digital signal representation :

→ Need of DSP :

(i) → To enhance the strength of the signal in communication

(ii) To employ complex mathematical operation of the signal in simple implementation.

→ Advantage of DSP

→ Application of DSP.

UNIT-II

DISCRETE TIME SYSTEM ANALYSIS :

Z-Transform :

The z transform of a discrete-time signal x(n) is

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\therefore X(z) = Z\{x(n)\}$$

$$x(n) \xleftrightarrow{Z} X(z)$$

Z-transform and ROC of Finite Duration Sequences

(i) Right hand seq

$x(n) = 0$ for all $n < n_0$, the ROC is entire z plane except $z=0$

(ii) Left Hand sequence : $x(n) = 0$ for all $n \geq n_0$.

(ROC is entire z-plane except at $z=\infty$)

(iii) Two-sided sequence

↳ A signal that has finite duration on both the left and right hand side (ROC is entire z-plane except at $z=0$, $z=\infty$)

Properties of the z-transform

(i) Linearity:

If $X_1(z) = Z\{x_1(n)\}$ & $X_2(z) = Z\{x_2(n)\}$

$$Z\{a x_1(n) + b x_2(n)\} = a X_1(z) + b X_2(z).$$

(ii) Time shift or translation:

(a) If $X(z) = Z\{x(n)\}$

$Z\{x(n-m)\} = z^{-m} X(z)$ $m \to$ is +ive or -ive integer

(b) If $X(z) = Z\{x(n)\}$

$$Z\{x(n+m)\} = z^{m} X(z).$$

(iii) Multiplication by an exponential sequence

If $X(z) = Z\{x(n)\}$ then

$$Z\{a^n x(n)\} = X\{a^{-1} z\}$$

(iv) Time Reversal:

If $X(z) = Z\{x(n)\}$

$$Z\{a^n x(n)\} = X(a^{-1} z^{-1})$$

(v) Differentiation of X(z)

If $X(z) = Z\{x(n)\}$ then

$$Z\{n x(n)\} = -z \frac{d}{dz} X(z).$$

(vi) Convolution theorem:

If $X(z) = Z\{x(n)\}$ & $H(z) = Z\{h(n)\}$ then

$$Z\{x(n) * h(n)\} = X(z) H(z)$$

(vii) Complex convolution theorem:

If $x_3(n) = x_1(n) x_2(n)$ then

$$X_3(z) = \frac{1}{2\pi j} \oint_c X_1(v) X_2\left(\frac{z}{v}\right) v^{-1} dv.$$

(ix) Parseval's theorem:

$$\sum_{n=-\infty}^{\infty} x_1(n) x_2^*(n) = \frac{1}{2\pi j} \oint_c X_1(v) X_2^*\left(\frac{1}{v^*}\right) v^{-1} dv.$$

(x) Initial value theorem :

If $X_+(z) = z\{x(n)\}$ then

$$x(0) = \lim_{z \to \alpha} X_+(z)$$

(xi) Final value theorem:

$$x(\alpha) = \lim_{z \to 1} (z-1)\, X_+(z)$$

(xii) correlation :

If $X_1(z) = z\{x_1(n)\}$ and $X_2(z) = z\{x_2(n)\}$ then

$$z[r_{x_1x_2}(l)] = z\left[\sum_{n=-\alpha}^{\alpha} x_1(n)\, x_2(n-l)\right] = \Gamma_{x_1x_2}(z)$$

$$= X_1(z)\, X_2(z^{-1})$$

Inverse Z-Transform

(i) Long Division Method

(ii) Partial fraction expansion Method

(iii) Residue Method

(iv) convolution Method

(i) Long Division Method!

$$X(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}}$$ = ge… diameter

(ii) Partial Fraction :

$$\frac{X(z)}{z} = \frac{c_1}{z-p_1} + \frac{c_1^*}{z-p_1^*}$$

(iii) Residue Method!

$x(n) = \sum$ [residues of $X(z)z^{n-1}$ at the poles inside c]

$$= \sum_i (z-z_i)\, X(z)\, z^{n-1}\Big|_{z=z_i}$$

(iv) convolution Method :

$$z[x_1(n) * x_2(n)] = X_1(z)\, X_2(z)$$

Application to discrete systems:

Stability Analysis:

A linear time-invariant system with the system function $H(z)$ is BIBO stable if and only if the ROC for $H(z)$ contain the unit circle. $|H(z)| \leq \sum_{n=0}^{\infty} |h(n)| < \infty$

Frequency Response:

$$|H(e^{j\omega})| = \sqrt{(Real)^2 + (Imag)^2}$$

$$\angle H(e^{j\omega}) = \theta = -\tan^{-1}\left(\frac{Imag}{Real}\right)$$

Difference Equation:

$$X(z) = \frac{b_0 + b_1 z^{-1} + \dots b_M z^{-M}}{1 + a_1 z^{-1} + \dots a_N z^{-1}}$$

Convolution:

$$x(n) = x_1(n) * x_2(n)$$

$$= \sum_{n=-\infty}^{\infty} x_1(n)\, x_2(n-k)$$

2 types
1) circular
   - concentric circle
   - Matrix method
2) linear convolution

If $x_1(n) \xleftrightarrow{z} X_1(z)$

$x_2(n) \xleftrightarrow{z} X_2(z)$

$\therefore \; x(n) = x_1(n) * x_2(n) \xleftrightarrow{z} X_1(z) * X_2(z) = X(z)$

Fourier transform of discrete sequence:

Discrete Fourier series:

$$x_p(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_p(k)\, e^{j2\pi kn/N}, \quad n = 0, 1, \dots N-1$$

$X_p(k)$, $k = 0, 1, \dots N-1$ are called Discrete Fourier series coefficient.

UNIT-III

DISCRETE FOURIER TRANSFORM & COMPUTATION

DFT Properties :

DFT Pair equation :

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N} \quad , k = 0,1,2 - N-1$$

IDFT :

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi nk/N} \quad , n = 0,1,2 -- N-1$$

$e^{-j\omega} = \cos\omega - j\sin\omega$

$e^{j\omega} = \cos\omega + j\sin\omega$

Properties :

(i) Periodicity : If the X(k) is an n-point DFT of x(n) then

$x(n+N) = x(n)$ for all n

$X(k+N) = X(k)$ for all k.

(2) Linearity :

$$a x_1(n) + b x_2(n) \xrightarrow[N]{DFT} a X_1(k) + b X_2(k)$$

(3) Shifting property :

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$ now shift the seq $x_p(n)$ by the k unit to the right

$$x_p(n-k) = \sum_{l=-\infty}^{\infty} x(n-lN-k)$$

(4) Convolution theorem :

$$x(n) = x_1(n) * x_2(n) \xrightarrow[N]{DFT} X_1(k) * X_2(k) = X(k)$$

(5) Time Reversal :

$$\text{If } x(n) \xrightarrow[N]{DFT} X(k)$$

$$x(-n \,l \bmod N) = x(N-n) \xrightarrow[N]{DFT} X(-k(\bmod N)) = X(N$$

(6) Circular Time shifting :

$$x(n) \xrightarrow[N]{DFT} X(k)$$

$$x(n - l(\bmod N)) \xrightarrow[N]{DFT} X(k)\, e^{-j2\pi k/N}$$

(7) circular frequency shifting:

$$x(n)\, e^{j2\pi l n/N} \xrightarrow[N]{DFT} X(K - l(\text{mod } N))$$

(8) complex conjugate property:

If $x(n) \xrightarrow[N]{DFT} X(K)$ then

$$x^*(n) \xrightarrow[N]{DFT} X^*(-K, (\text{Mod}(N))) = X^*(N-K)$$

Hence $x^*(-n(\text{mod } N)) = x^*(N-n) \xrightarrow[N]{DFT} X^*(K)$

(9) circular convolution:

If $x_1(n) \xrightarrow[N]{DFT} X_1(K)$ and

$x_2(n) \xrightarrow[N]{DFT} X_2(K)$ then

$$x_1(n) \circledN x_2(n) \xrightarrow[N]{DFT} X_1(K) X_2(K)$$

(10) multiplication of two sequence:

$$x_1(n) x_2(n) \xrightarrow[N]{DFT} \frac{1}{N} X_1(K) \circledN X_2(K)$$

11) parseval theorem:

$x(n) \xrightarrow[N]{DFT} X(K)$

$y(n) \xrightarrow[N]{DFT} Y(K)$

$$\sum_{n=0}^{N-1} x(n) y^*(n) = \frac{1}{N} \sum_{n=0}^{N-1} X(K) Y^*(K)$$

$x(n) = y(n)$

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |X(K)|^2$$

12) circular convolution

$$r_{xy}(l) \xrightarrow[N]{DFT} \frac{1}{N} R_{xy}(K) = X(K) Y^*(K)$$

Magnitude and phase representation:

$$\text{Magnitude} = \sqrt{(\text{real})^2 + (\text{imag})^2}$$

$$\text{Phase} \quad \angle X(K) = \theta(K) = -\tan^{-1}\left(\frac{\text{ima}}{\text{real}}\right)$$

computation of DFT using FFT algorithm:

DFT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nK/N}, \quad K = 0, 1, 2 \ldots N-1$$

$$W_N = e^{-j2\pi N}$$

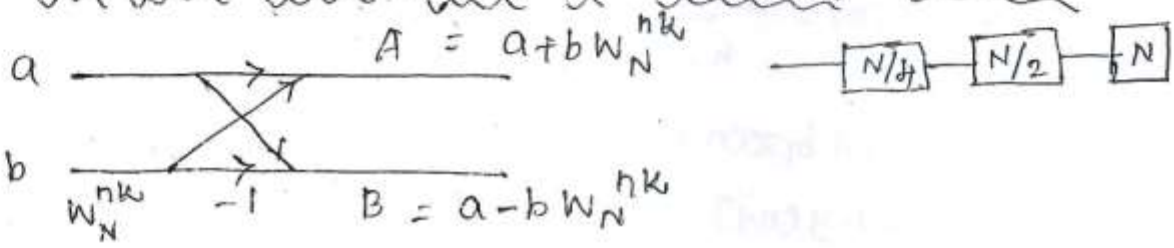
$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nK} \qquad K = 0, 1, 2 \ldots N-1$$

IDFT

$$\bar{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-nk} \qquad n = 0, 1, 2 \ldots N-1$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi nk/N}$$

DIT-FFT using radix-2 Butterfly structure:



DIT FFT → DFT, IDFT

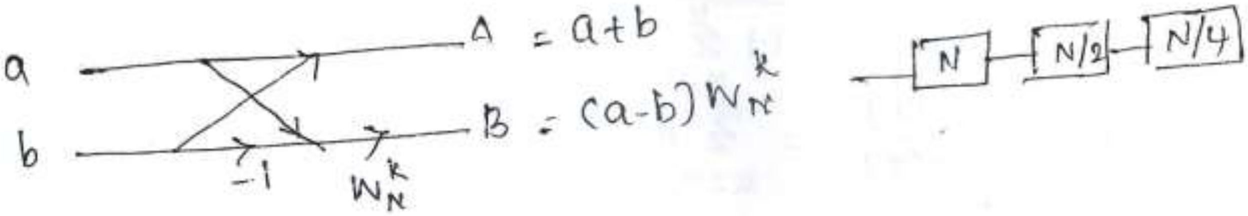
DFT:

(i) Input should be in bit reverse
(ii) Output is sequence
(iii) $x(n) \to$ Given
$X(k) \to$ Find
(iv) $W_N^{k} = e^{-j2\pi k/N}$

IDFT

(i) Input is ~~sequence~~ bit reversal
(ii) Output is bit ~~reversal~~ sequence
(iii) Output should be divided by 'N'
(iv) $X(k) \to$ Given, $x(n) \to$ Find
(v) Twiddle factor $W_N^{-k} = e^{j2\pi k/N}$.

DIF-FFT using radix-2 Butterfly structure:
(DIF - Decimation In Frequency)



DIF-FFT → DFT, IDFT (same Procedure)

DFT

1) Input should be sequence
2) Output is bit reversal
3) $x(n) \to$ given, $X(k) \to$ Find
4) $W_N^{k} = e^{-j2\pi k/N}$

IDFT

(1) Input → sequence
(2) Output → bit reversal
(3) $x(n) \to$ Find,
$X(k) \to$ Given
(4) Output should be divided by 'N'
(5) Twiddle Factor
$W_N^{-k} = e^{j2\pi k/N}$

UNIT-IV

DESIGN OF DIGITAL FILTERS

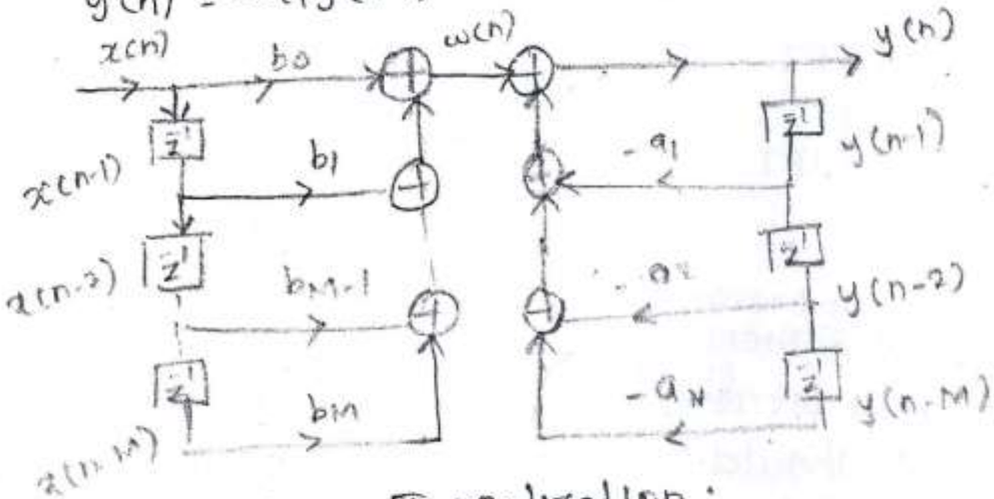
IIR Realization :

(i) Direct Form-I realization

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$b_0 x(n) + b_1 x(n-1) + \cdots b_M x(n-M) = w(n) \rightarrow ①$$

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) \cdots - a_N y(n-N) + w(n) \rightarrow ②$$

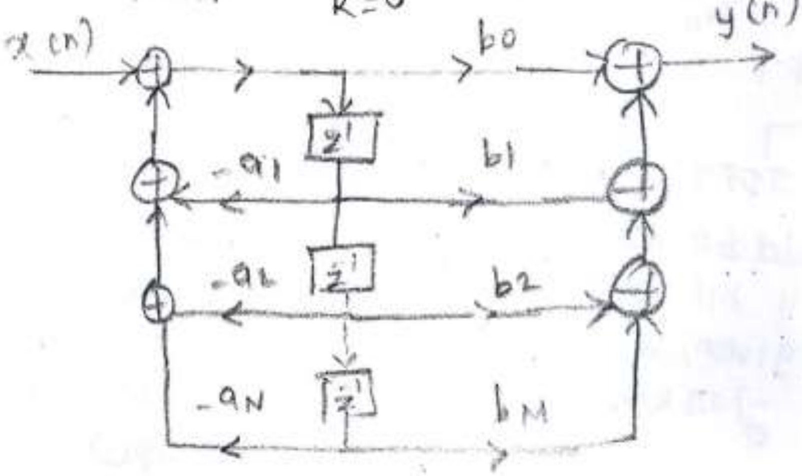


(ii) Direct Form-II realization :

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = \frac{Y(z)}{X(z)} \cdot \frac{W(z)}{W(z)}$$

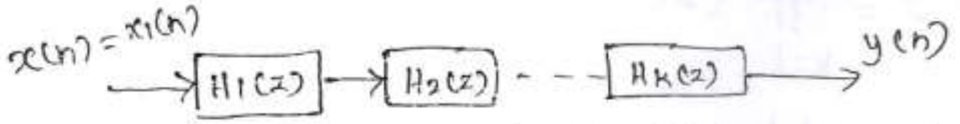
$$\frac{W(z)}{X(z)} = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}} \quad ; \quad W(z) = X(z) - a_1 z^{-1} W(z) \cdots - a_N z^{-N} W(z)$$

$$\frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k} \quad ; \quad Y(z) = b_0 W(z) + b_1 z^{-1} W(z) \cdots b_M z^{-M} W(z)$$



(iii) Cascade Form :

$$H(z) = H_1(z)\, H_2(z) \,\ldots\, H_K(z)$$



$$H(z) = \frac{(b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2})\,(b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2})}{(1 + a_{k1}z^{-1} + a_{k2}z^{-2})\,(1 + a_{m1}z^{-1} + a_{m2}z^{-2})}$$

$$H_1(z) = \frac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$$

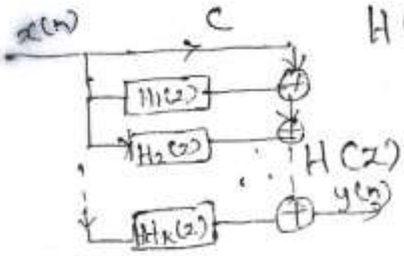
$$H_2(z) = \frac{b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2}}{1 + a_{m1}z^{-1} + a_{m2}z^{-2}}$$

By using Direct Form - II and cascading.

(iv) Parallel Form :

→ It can be obtained by performing a parallel Expansion of

$$H(z) = c + \sum_{k=1}^{N} \frac{c_k}{1 - p_k z^{-1}} \qquad p_k \to \text{Poles of } H(z).$$



$$\therefore H(z) = c + \frac{c_1}{1 - p_1 z^{-1}} + \frac{c_2}{1 - p_2 z^{-1}} + \ldots \frac{c_N}{1 - p_N z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = c + H_1(z) + H_2(z) \ldots H_N(z)$$

$$Y(z) = cX(z) + H_1(z)\, X(z) + \ldots H_N(z)\, X(z)$$

$c \Rightarrow$ can be caculated by using long division method

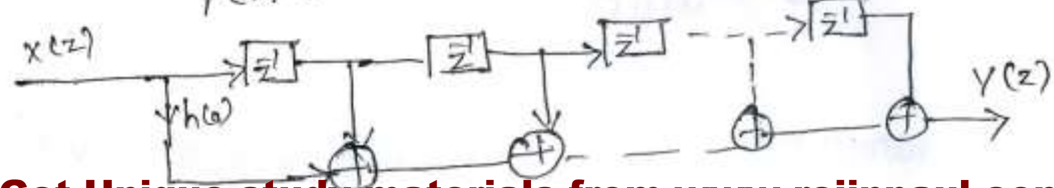
FIR Realization :

(i) Transversal Structure.

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$\frac{Y(z)}{X(z)} = h(0) + h(1)z^{-1} + \ldots h(N-1)z^{-(N-1)}$$

$$Y(z) = h(0)X(z) + h(1)X(z)z^{-1} \ldots h(N-1)z^{-(N-1)}X(z)$$

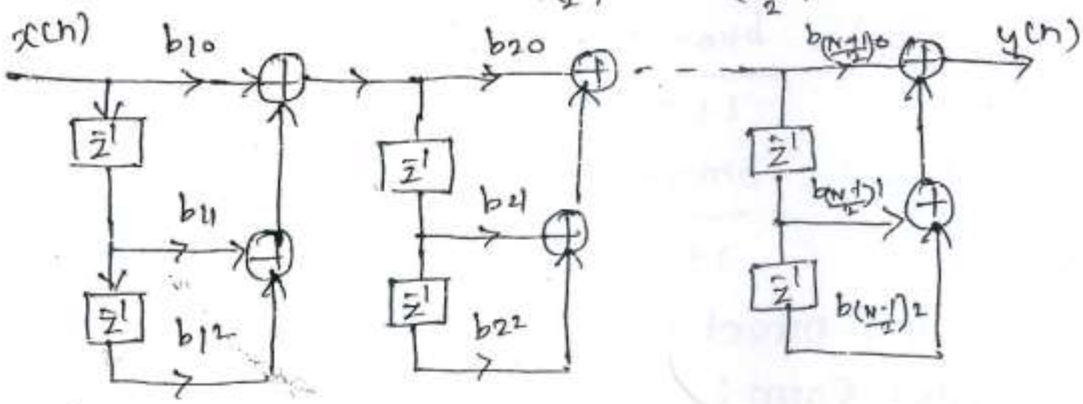


Cascade Realization:

$$H(z) = \prod_{k=1}^{\frac{N-1}{2}} (b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2})$$

For N = odd

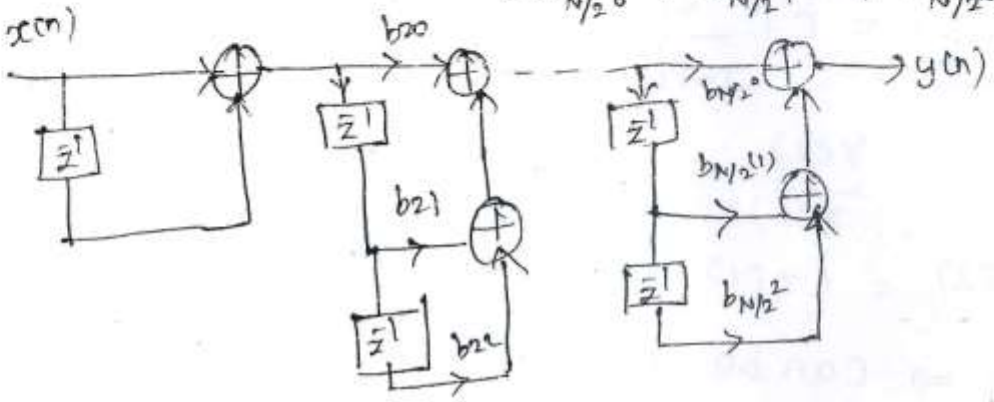
$$H(z) = (b_{10} + b_{11} z^{-1} + b_{12} z^{-2})(b_{20} + b_{21} z^{-1} + \ldots) \ldots (b_{(\frac{N-1}{2})0} + b_{(\frac{N-1}{2})} z^{-1} + b_{(\frac{N-1}{2})} z^{-2})$$



For N = even

$$H(z) = (b_{10} + b_{11} z^{-1}) \prod_{k=2}^{N/2} (b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2})$$

$$H(z) = (b_{10} + b_{11} z^{-1})(b_{20} + b_{21} z^{-1} + b_{22} z^{-2})(b_{30} + b_{31} z^{-1} + b_{32} z^{-2}) \ldots (b_{N/2\,0} + b_{N/2\,1} z^{-1} + b_{N/2\,2} z^{-2})$$



FIR Design using windowing Techniques:

1. Choose the desired freq. response $H_d(e^{j\omega})$
2. Find $h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$
3. Choose the window

(i) Hanning $\quad w_H(n) = 0.5 + 0.5 \cos 2\pi n / N-1 \quad$ For $-\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$

$0 \quad$ otherwise

(ii) Hamming $\quad w_H(n) = 0.54 + 0.46 \cos(2\pi n / N-1) \quad$ for $-\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$

(iii) Rectangular window

$$w_R(n) = 1 \quad \text{for} \quad -\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2}$$
$$0 \quad \text{otherwise}$$

(iv) Triangular or Bartlett window

$$w_T(n) = 1 - \frac{2|n|}{N-1} \quad \text{for} \quad -\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2}$$

(v) Raised cosine window

$$\alpha + (1-\alpha)\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2}$$

$\alpha = 0.5$ for Hanning

$\alpha = 0.54$ For Hamming

(vi) Blackman window

$$w_B(n) = 0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}$$

for $-\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2}$

(continuation steps in page 17)

Need and choice of window:

$$H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-j\omega n}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

→ one possible way of obtaining FIR Filter is to truncate the infinite Fourier series at $n = \pm\left(\frac{N-1}{2}\right)$

N → length of the desired seq.

→ But the truncation of the fourier series result in the oscillation in Passband and stopband.

→ The oscillations are due to slow convergence of the Fourier series and this effect is known as the Gibb's Phenomenon.

Ideal Frequency Response:

(i) Low Pass

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; -\omega_c \le \omega \le \omega_c \\ 0 & ; -\pi \le \omega \le -\omega_c \\ 0 & ; \omega_c < \omega < \pi \end{cases}$$

(ii) High Pass

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; -\pi \le \omega \le -\omega_c \\ e^{-j\omega\alpha} & ; \omega_c \le \omega \le \pi \\ 0 & ; -\omega_c < \omega < \omega \end{cases}$$

(iii) Band Pass

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; -\omega_{c2} < \omega \le -\omega_{c1} \\ e^{-j\omega\alpha} & ; \omega_{c1} \le \omega \le \omega_{c2} \\ 0 & ; -\pi \le \omega \le -\omega_{c2} \\ 0 & ; -\omega_{c1} < \omega < \omega_{c1} \\ 0 & ; \omega_{c2} < \omega \le \pi \end{cases}$$

(iv) Bandstop

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; -\pi \le \omega \le -\omega_{c2} \\ e^{-j\omega\alpha} & ; -\omega_{c1} \le \omega \le \omega_{c1} \\ e^{-j\omega\alpha} & ; \omega_{c2} \le \omega \le \pi \\ 0 & ; -\omega_{c2} < \omega < \omega_{c1} \\ 0 & ; \omega_{c1} < \omega < \omega_{c2} \end{cases}$$

Linear Phase Characteristics:

→ The transfer function of FIR Causal filter is

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \rightarrow ①$$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

The above equation is periodic with period 2π

$$H(e^{j\omega}) = \pm |H(e^{j\omega})| e^{j\theta(\omega)}$$

$|H(e^{j\omega})|$ – Magnitude

$e^{j\theta(\omega)}$ – Phase with period of 2π

(4) Find the Filter coefficients from $h_d(n)$

(i.e.) $h_d(0), h_d(1)$ --- for $-\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$

(5) Find the window coefficients from $w_H(n)$

(i.e.) $w_H(0), w_H(1)$ --- for $-\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2}$

(6) Find the filter coefficients using window.

$$h(n) = h_d(n)\, w_H(n)$$

(i.e.) $h(0), h(1)$ ---- $-\left(\frac{N-1}{2}\right) \le n \le \frac{N-1}{2}$

(7) Find the transfer function of the filter

$$H(z) = h(0) + \sum_{n=1}^{N-1/2} h(n)\left[z^{-n} + z^{n}\right] \qquad -\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$$

(8) Find the transfer function of realizable filter

$$H'(z) = z^{-(N-1/2)}\, H(z)$$

From this find $h(0), h(1), h(2)$ ---- $h(N)$

(9) Find the magnitude Response

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{N-1/2} a(n) \cos \omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right)$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

Analog Filter Design :

(i) Butterworth :

(i) Find the order of the filter N

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log(\Omega_s/\Omega_p)}$$

$$\lambda = (10^{0.1\alpha_s}-1)^{0.5}, \quad \varepsilon = (10^{0.1\alpha_p}-1)^{0.5}$$

$$A = \lambda/\varepsilon, \quad 1/k = \frac{\Omega_s}{\Omega_p}$$

$$N \geq \frac{\log(\lambda/\varepsilon)}{\log(1/k)}$$

$$N \geq \frac{\log A}{\log(1/k)}$$

(ii) ~~Truncate~~ Round off the order to the next High Integer.

(iii) IF N is odd (Find Poles)

$$s_k = e^{j\pi k/N} \quad ; \quad k = 1, 2 --- 2N$$

IF N is even

$$s_k = e^{j\varphi_k}$$

$$\varphi_k = \pi/2 + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, -- N$$

(iv) Find $H(s) = \frac{1}{\text{Poles}}$ when $\Omega_c = 1$ rad/sec.

(v) Find $\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}}$ and sub $s = \frac{s}{\Omega_c}$ in the transfer function $H(s)$.

For HPF $\Omega_c = \Omega_p$, & $s = \frac{\Omega_c}{s}$

Analog Lowpass Chebyshev Filter:

1. From the given specifications find the order of the filter N

$$N \geq \frac{\cosh^{-1}(\lambda/\varepsilon)}{\cosh^{-1}\frac{\Omega_s}{\Omega_p}}$$

$$\varepsilon = (10^{0.1\alpha_p} - 1)^{0.5}$$
$$\lambda = (10^{0.1\alpha_s} - 1)^{0.5}$$

2. Round off it to the next Higher Integer.
3. Using the following formulas find the values of a and b

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

where $\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2} + 1}$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

4. $s_k = a\cos\phi_k + jb\sin\phi_k$

$$\phi_k = \pi/2 + \left(\frac{2k-1}{2N}\right)\pi \quad , k = 1, 2 \ldots N$$

5. Find the denominator polynomial of the transfer function using the above poles.
6. The numerator of the transfer function depends on the value of N.

(a) For N is odd sub $s=0$ in the denominator polynomial & find the value. This value is equal to the numerator of the transfer function

(b) For N is even sub $s=0$ in the denominator polynomial & divide the result by $\sqrt{1+\varepsilon^2}$. This value is equal to the numerator

Digital Design using Impulse Invariant

$$H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k} \rightarrow (1)$$

then $H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_k T} z^{-1}}$

Using Bilinear Transformation

$$H(z) = H(s) \Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

sub $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$ to obtain $H(z)$.

Warping Effect :

Let Ω and ω represent the freq variables in the analog filter and the desired digital filter respectively.

$$\Omega = \frac{2}{T} \tan \omega/2$$

Prewarping :

$$\Omega = \frac{2}{T} \tan \omega/2$$

For High Pass

$$\Omega_p = \frac{2}{T} \tan \omega_p/2 \qquad \omega_p = 2\pi f_p.$$

$$\Omega_s = \frac{2}{T} \tan \omega_s/2. \qquad \omega_s = 2\pi f_s.$$

Low pass

$$\Omega_p = 2\pi f_p, \quad \Omega_s = 2\pi f_s.$$

Frequency Transformation :

(i) Lowpass to Lowpass

$$z^{-1} = \frac{z^{-1}-\alpha}{1-\alpha z^{-1}}$$

where $\alpha = \frac{\sin[(\omega_p-\omega_p')/2]}{\sin[(\omega_p+\omega_p')/2]}$

(ii) Lowpass to highpass

$$z^{-1} = -\left[\frac{z^{-1}+\alpha}{1+\alpha z^{-1}}\right]$$

$$\alpha = \frac{\cos[(\omega_p' + \omega_p)/2]}{\cos[(\omega_p' - \omega_p)/2]}$$

$\omega_p \rightarrow$ Passband freq of LPF

$\omega_p' \rightarrow$ " " of HPF

(iii) Lowpass to Bandpass

$$z^{-1} = \frac{-\left[z^{-2} - \frac{2\alpha k}{1+k} z^{-1} + \frac{k-1}{k+1}\right]}{\frac{k-1}{k+1} z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + 1}$$

where $\alpha = \dfrac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$

$k = \cot\left[\dfrac{\omega_u - \omega_l}{2}\right] \tan\dfrac{\omega_p}{2}$

$\omega_u \rightarrow$ upper cutoff freq, $\omega_l \rightarrow$ lower cutoff freq

(iv) Lowpass to Band stop:

$$z^{-1} = \frac{z^{-2} - \frac{2\alpha}{1+k} z^{-1} + \frac{1-k}{1+k}}{\frac{1-k}{1+k} z^{-2} - \frac{2\alpha}{1+k} z^{-1} + 1}$$

where $\alpha = \dfrac{\cos[(\omega_u + \omega_l)/2]}{\cos[(\omega_u - \omega_l)/2]}$

$k = \tan[(\omega_u - \omega_l)/2] \tan\dfrac{\omega_p}{2}$.

UNIT-V

DIGITAL SIGNAL PROCESSORS

Introduction :

(i) General purpose digital signal Processor

→ They include Fixed point processors such as Texas Instruments

eg : TMS320C5X, TMS320C54X.

(ii) Special purpose digital signal Processor

→ Designed for specific DSP algorithm such as FFT

→ Hardware designed for specific application such as PCM and Filtering.

Architecture :

(i) Von Neumann Architecture

(ii) Harvard Architecture

(iii) VLIW Architecture

(iv) Multiply Accumulate UNIT (MAC)

(v) Pipelining

Features :

(i) Architectural features

(ii) Execution speed

(iii) Type of arithmetic

(iv) word length.

Addressing Modes :

(i) Immediate Addressing

(ii) Indirect Addressing

(iii) Register Addressing

(iv) Memory Mapped register addressing

(v) Direct addressing

(vi) circular addressing mode.

Functional mode and Instruction set:

(i) Accumulator memory Reference Instruction

ABS → Absolute value of Acc.

ADCB → Add AccB and carry bit to Acc.

(ii) Parallel Logic unit Instruction

APL → AND Data memory value with DBMR and store result in datamemory

CPL → Compare data memory value with DBMR

(iii) Branch and call Instruction

BACC → Branch to pgm memory location specified by ACCL.

BD → Delayed branch conditionally to pgm memory location.

CALA → Call to subroutine addressed by ACCL

(iv) I/O and Data memory operation Instruction

BLDP → Block move from data to pgm memory with destination address in BMAR

DMN → move data in data memory

IN → input data from I/O Port todata memory location.

(v) control Instruction

BIT → Test bit

NOP → No operation

POP → Pop top of stack to ACCL

PUSH → Push ACCL to top of stack

vi) Arithmetic Instruction

→ ADCB
→ SBB

vii) Logical Instruction

→ AND
→ OR, XOR

viii) Shift Instruction

ROL → Rotate accumulator left
ROR → " " Right.

ix) Load and Store Instruction

LACB → Load accumulator with ACCB
LACC → Load accumulator with shift

x) Move Instruction

DMOV → Data move in data memory

xi) Branch and call Instruction

BACC → Branch to location specified by accumulator

BANZ → Branch on auxilliary register not ze

Introduction to commercial processor:

→ TMS320C50 :
→ Bus structure
→ central Processing unit
→ on chip memory
→ on-chip peripherals.